IASDR07
INTERNATIONAL ASSOCIATION OF SOCIETIES OF DESIGN RESEARCH THE HONG KONG POLYTECHNIC UNIVERSITY 12TH TO 15TH NOVEMBER 2007

BEYOND THE BILATERAL - SYMMETRY IN TWO-DIMENSIONAL DESIGN

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ABSTRACT:

This paper identifies various geometric concepts, principles and constructions which are of great value to design researchers and practitioners. Particular attention is focused on geometric symmetry as the basis of an analytical tool to examine designs in different cultural or historical contexts. Relevant empirical literature is identified. Illustrations of regularly repeating designs are presented; these result from the application of a system of pattern construction, based on taking components of certain regular polygons and applying various symmetry operations.

Key words: geometry, symmetry, pattern

1. INTRODUCTION

Knowledge of many of the geometric principles, concepts and perspectives underlying structure and form in design can be sourced in ancient times. From the time of Euclid (c.300 BCE), until recently, geometry was the tool of choice for engineers, builders, artists and designers. Surprisingly, the foundations of such knowledge have been largely absent from the design curriculum worldwide and basic geometry, as once taught, has been unfashionable among educationists for several decades. Possibly as a result of this, there appears to be a widespread lack of understanding of the true potential of geometry as a design tool, and a lack of appreciation of its usefulness also as a research tool. In fact, the application and actualisation of geometry in all design disciplines is a key issue which should be of concern to both practitioners and analysts. In recent years, there have been some indications that the tide is changing and, in the wake of the publication of certain new text-books, or the availability of (and demand for) editions of relatively well-known text-books, design geometry and geometrical analysis appear to be receiving more attention from scholars and design educationists (Kappraff, 1986; Pearce, 1978; Elam, 2001; Leborg, 2006). It will however take some time before this apparent interest in design geometry feeds directly into the design curriculum and before geometrical analysis receives widespread attention from design researchers. It is the contention of the authors that an understanding of the basic principles of Euclidean geometry can offer immense potential in addressing and solving design problems in the twenty-first century. Also, geometric analysis of two and three-dimensional designs is reproducible from one analyst to another. Designs may be created and developed by reference to structural rules and, subsequently, they may be analyzed with respect to their underlying structural characteristics. The geometric analysis of representative groups of designs, can uncover a wide range of social, psychological, philosophical and cultural properties or characteristics. This was highlighted, during the 1980s and 1990s by scholars such as Washburn and Crowe [1988], who did much in the cause of promoting the potential of symmetry analysis in cultural contexts. The wider spectrum of geometric analysis was considered in an interesting article by Reynolds [2001]. Various geometric characteristics, principles, concepts, constructions, comparative measures and ratios are of particular importance to both the design practitioner and the design analyst. These include the following:

- 1:1 (square).
- π: radius (circle).
- Square root series $\sqrt{2}$ (=1.4142): 1; $\sqrt{3}$ (=1.732): 1; $\sqrt{4}$ (=2): 1 etc.
- Regular polygons (particularly squares, pentagons and hexagons), Reauleaux polygons, the ad quadratum, the vesica pisces, the sacred cut and other constructions.
- The golden section, Phi (ϕ) or 1.618:1 and various associated constructions such as the golden rectangle or golden spiral.
- Triangles (equilateral, isosceles, right angle, scalene).
- Lattice structures (including Bravais lattices) and grids based on the Platonic, Archimedean or other sorts of tilings.
- Various musical series, including 1:1; 1:2; 2:3; 3:4, etc.

- Geometric symmetry and its component geometric operations (or symmetries).

It has been argued elsewhere that all of the above are of value in the armoury of both the practitioner and the analyst, and that design geometry should form an important component of the design curriculum (Hann and Thomas, 2007). However, this paper focuses on only one of the areas listed: geometric symmetry. The intention is to explain the nature of geometric symmetry, to highlight its potential value as an analytical tool to the design researcher and to show how an understanding of the relevant fundamental principles can also aid the design practitioner.

2. SYMMETRY – A REVIEW OF BASIC PRINCIPLES

Symmetry pervades our everyday lives and environment. We live in a symmetrical world. We wear clothes which are symmetrical. We live and work in buildings which are largely symmetrical. We drive automobiles which are symmetrical. In fact the vast majority of living creatures, manufactured objects, constructions, monuments, tools, implements and utensils exhibit bi-lateral symmetry. This is where two component and equal parts are each a reflection of the other. The meaning of the term symmetry can be extended beyond this every-day use to include other geometrical actions and their combinations; in all cases the essence is one of regular reproduction or repetition of a fundamental unit, shape, figure or other element. These further geometric actions are known as "symmetry operations" or "symmetries", and are most readily understood if considered in a two-dimensional context, although it should be noted that many of the relevant concepts were developed in association with the improved understanding of three-dimensional crystal structures in the late-nineteenth and twentieth centuries.

Four symmetry operations are of importance in the context of two-dimensional design: translation, rotation, reflection and glide-reflection (shown schematically in Figure 1).

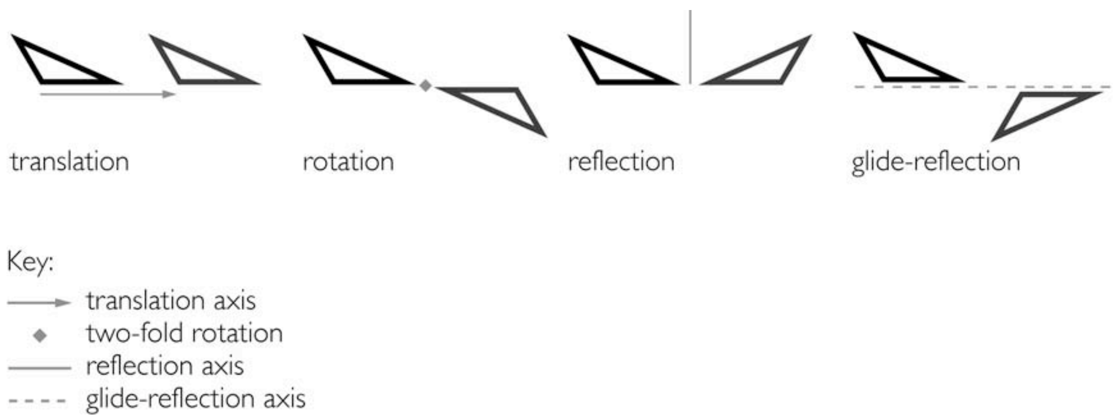

Figure 1: Schematic illustrations of the four symmetry operations.

Translation allows a motif to undergo repetition vertically, horizontally, or diagonally at regular intervals while retaining the same orientation. Rotation allows a motif to undergo repetition at regular intervals round an imaginary fixed point (known as a centre of rotation). Reflection allows a motif to undergo repetition across an imaginary line, known as a reflection axis, producing a mirror image; this is characteristic of so-called bilateral symmetry. Glide-reflection allows a motif to be repeated in one action through a combination of translation and reflection, in association with a glide-reflection axis. Where motifs or patterns possess the same symmetry characteristics they are said to be of the same class, and may be classified accordingly; a full explanation was given by Hann and Thomson (1992).

Motifs are the building blocks from which patterns are produced. They may be either symmetrical or asymmetrical. A symmetrical motif is comprised of two or more parts, of identical size, shape and content. Depending on the constituent symmetry characteristics, motifs may be classified using the notation cn (c for cyclic) or dn (d for dihedral). Motifs from family cn have n-fold rotational symmetry and motifs from family dn have n distinct reflection axes as well as n-fold rotational symmetry. Relevant illustrations are provided in Figures 2 and 3.

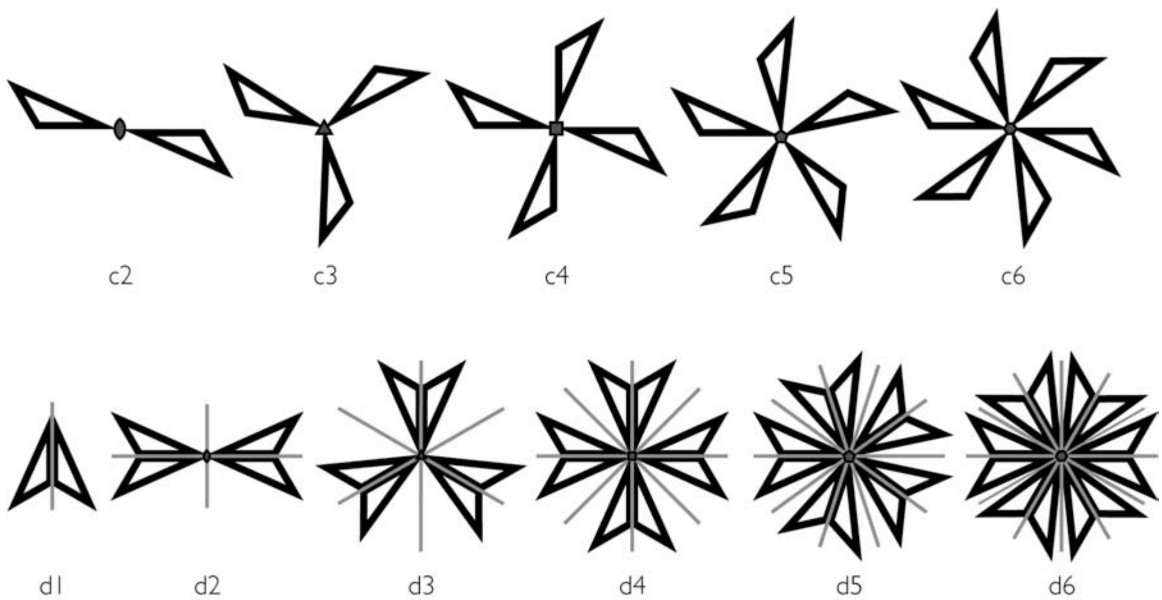

Figure 2: Schematic illustrations of classes cn and dn motifs.

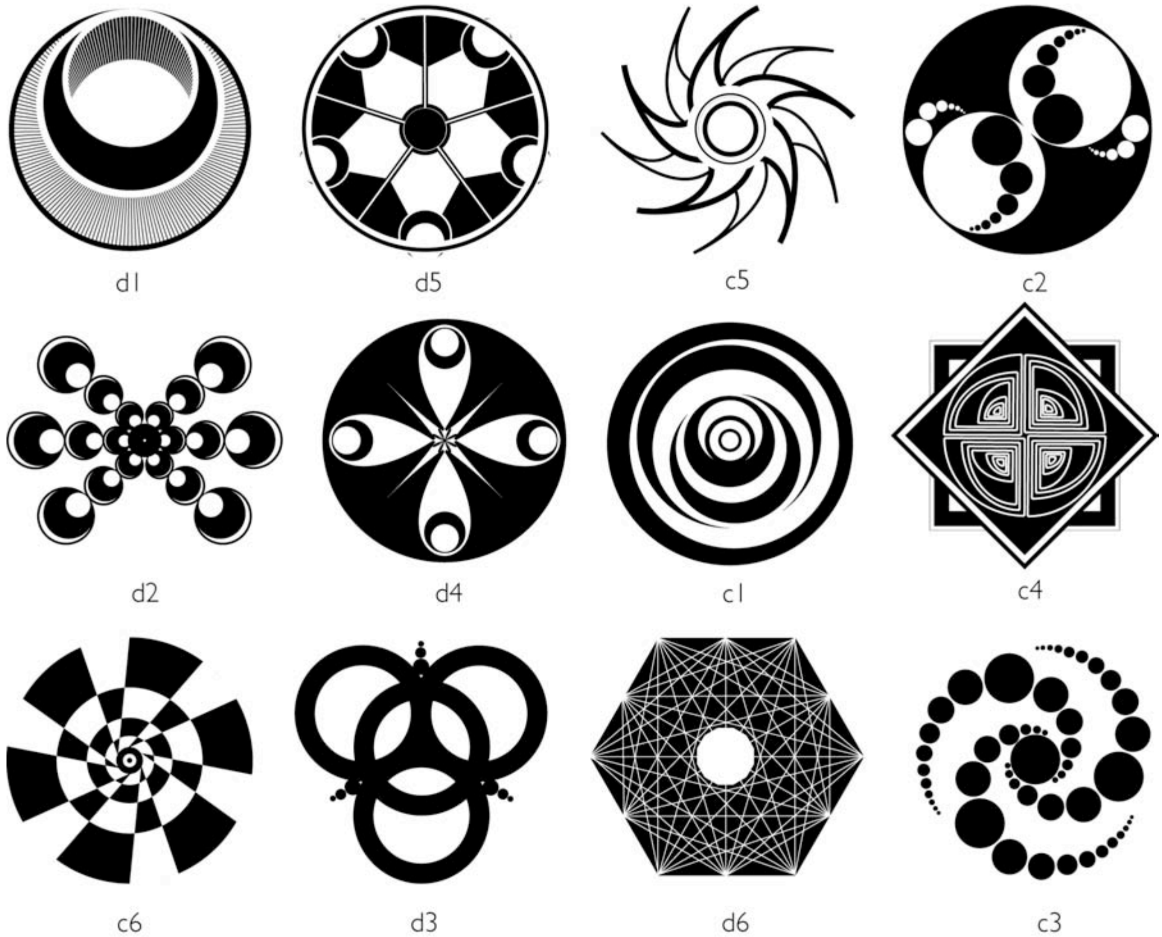

Figure 3: Illustrations of classes cn and dn motifs.

The term pattern is frequently used to refer to surface variation or texture. More precisely a pattern exhibits an underlying regular structure, showing repetition of a motif, figure or other unit. In considering the geometry of pattern, the term "symmetry" may be introduced. Border patterns exhibit translation of a motif at regular intervals in one direction only, as if trapped between two imaginary parallel lines. Alternative terms include band, strip, frieze or one-dimensional patterns. Combinations of the four symmetry operations yield seven possible border pattern classes (shown in Figures 4 and 5). The notation conventionally ascribed to border patterns is of the form pxyz. The letter p prefaces each of the seven. The letter x is the symbol which denotes symmetry operations perpendicular to the longitudinal axis of the border; m is used where vertical reflection is present, or the number 1 where the operation is absent. The third symbol, y, denotes symmetry operations working parallel to the sides of the border; the letter m is used if longitudinal reflection is present, the letter a if glide reflection is present or the number 1 if neither is present. The fourth symbol, z, denotes the presence of two-fold rotation; the number 2 is used if rotation is present and the number 1 if rotation is not present.

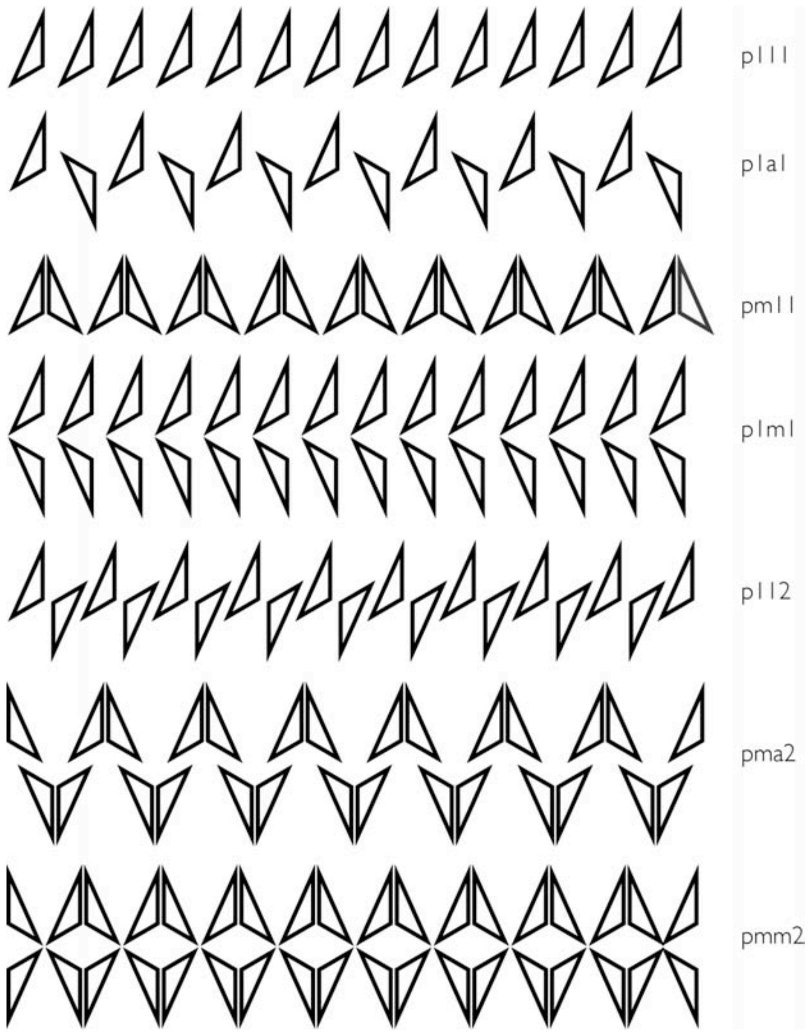

Figure 4: Schematic illustrations of the seven classes of border patterns.

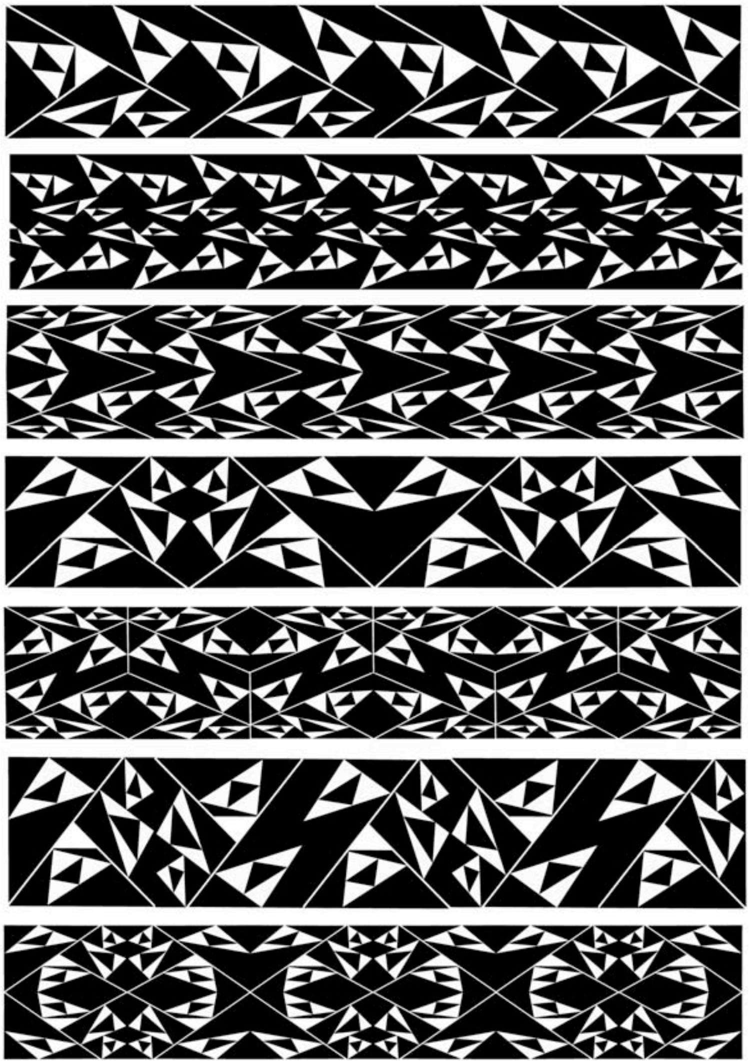

Figure 5: Illustrations of the seven classes of border patterns.

All-over patterns are characterized by translation in two independent directions across the plane. Combinations of the four symmetry operations will yield seventeen possibilities (or classes). Associated with these seventeen classes is a notation, which identifies the highest order of rotation within the pattern together with the presence (or absence) of glide-reflection and/or reflection. It should be stressed that there are only seventeen classes (referred to as "primary classes of all-over patterns"). Where rotation is present, it may be of the order two, three, four or six, and some patterns may show combinations of these. Reflection, where it is present, may be in one or more directions, and may combine with other symmetry operations; the same is true of glide-reflection. The seventeen all-over pattern classes are illustrated in Figure 6.

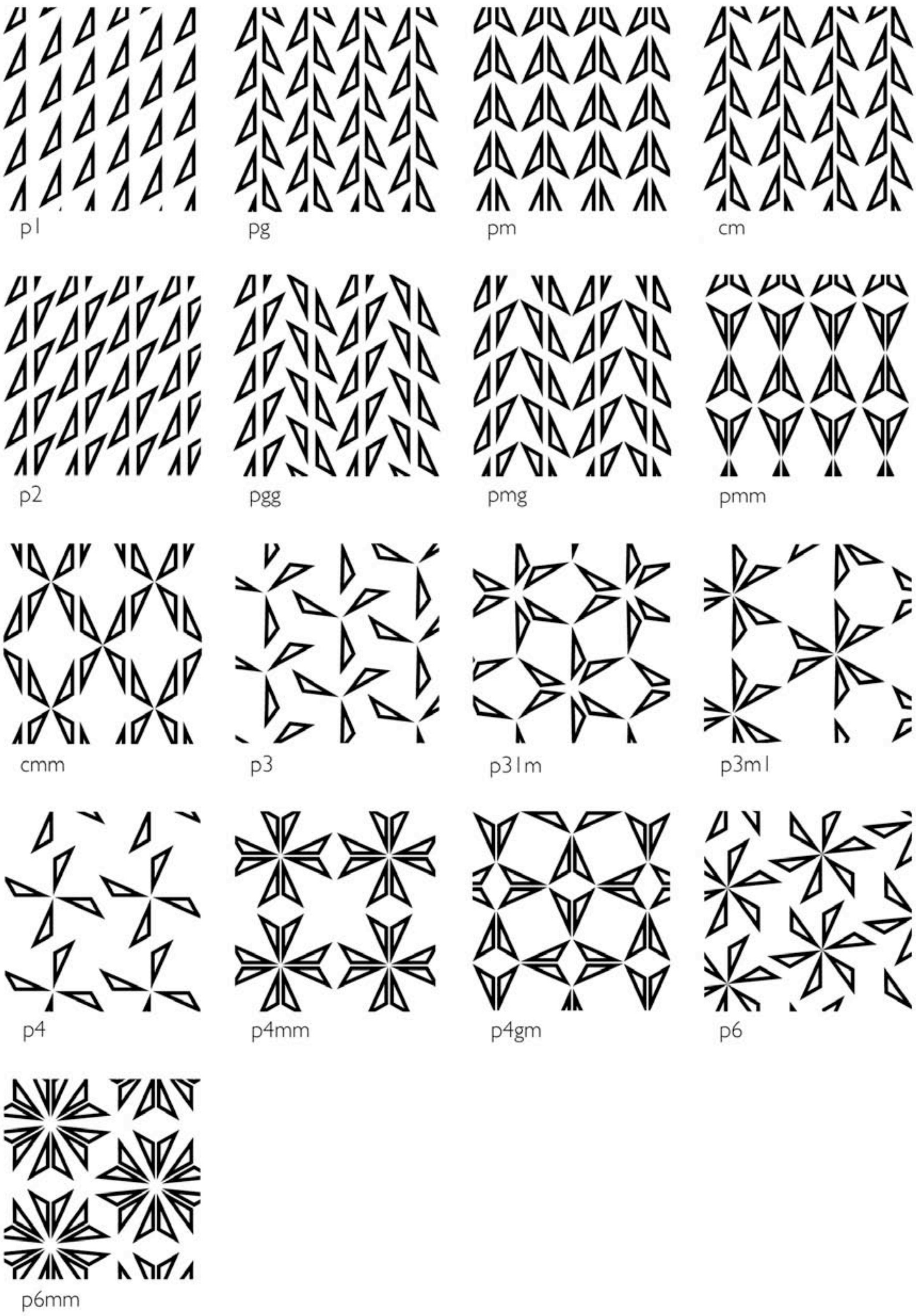

Figure 6: Schematic illustrations of the seventeen classes of all-over patterns.

3. DESIGN ANALYSIS

For much of the twentieth century design historians and theorists, as well as anthropologists and archaeologists, restricted their studies of two-dimensional design to broad-ranging subjective commentary and superficial analysis; cross-cultural considerations and comparisons were hindered generally by the apparent lack of awareness of a procedure to systematically classify two-dimensional designs in a way which was both meaningful and reproducible. Definitive classification is fundamental where research is to be supplemented and extended. The use of a classification system which lends itself to the precise ordering of data has ramifications of importance to anthropologists, archaeologists and design historians. Uniform classification makes possible: the systematic study of data; the replication of results among researchers; the formation of plausible hypotheses and explanations; cross-cultural considerations and comparisons. In the context of figures, motifs and regularly repeating patterns, a uniform means of classification can be clearly charted. This system evolved over the course of the twentieth century and is based on considering the symmetry (or geometrical) characteristics of designs.

It was seen above that motifs can be either symmetrical or asymmetrical, and that symmetrical motifs can have either rotational and/or reflection characteristics. Also, border patterns can be grouped into one of seven types and all-over patterns into one of seventeen types. In the latter part of the twentieth century, various empirical studies (conducted mainly be archaeologists, mathematicians or cultural anthropologists) set out to classify designs from different cultural settings and historical periods by reference to their symmetry characteristics. Notable studies include: Shepard (1948) who showed that times of cultural change could be pinpointed by using symmetry classification (in that the symmetry preferences will change as cultural changes take root). Crowe (1971, 1975, and 1982) conducted a range of studies of African decorative art; in each case different symmetry classes were found to predominate. Also, changes in pattern preferences over time were accounted for by parallel changes in society. Zaslow and Dittert (1977) found a connection between various social factors and the selection of patterns with particular symmetry classes. Asher and Asher (1981), in their study of Inca society, found that various aspects of culture were associated with dominant design structures (e.g. structure of residential compound). Van Esterik (1979) made a study of pottery designs from the site of Ban Chiang (Thailand) and found that particular pattern classes were consistently preferred. Kent (1983), in a study of prehistoric textiles from the American Southwest, found a correlation between changes in design structure and other major events in the chronology of the area. Washburn (1977, 1983, 1984 and 1986) presented a series of studies: Anasazi ceramics, Greek Neolithic ceramics and Indian basketry (California). Social causes for selection and change of dominant pattern types were highlighted in each case. Campbell (1989) from his study of Pueblo pottery concluded that symmetry classification provided an easy comparison of artifacts from different sites. Hann (1992 and 1993) conducted a range of studies: traditional Javanese batiks, traditional

Sindhi ajarak patterns, Jacquard woven French silks and Japanese textile patterns from the Edo period (1604 – 1867). Two important conclusions were reached. First, when the patterns from a particular culture were classified with respect to their symmetry characteristics (i.e. into the seven classes of border patterns or the seventeen classes of all-over patterns) non-random distributions were found in each case. This indicated that design types (judged in terms of geometric symmetry) are not selected randomly and that symmetry classification is a culturally sensitive tool. Second, the symmetry preferences of a culture will remain broadly similar in the absence of outside pressures for change. Where outside influences were evident (often communicated through trade) and apparent changes in the culture took place, symmetry preferences also had a tendency to change. So, cultural change is reflected in changes in symmetry preferences. Subject to the availability of representative data, symmetry classification can thus be used to indicate cultural preferences, continuity and change. It is the contention of the authors that this relationship between geometry and culture is not just expressed through one particular geometric feature (i.e. symmetry) and that the other geometric measures, ratios or proportions listed in the introduction may also prove of value in the analysis of design. [This is the subject of a current research project being conducted by the authors]. A relatively recent review of conceptual developments in the study of geometric symmetry was provided by Hann (2003).

4. DESIGN SYNTHESIS

Although designers working in two dimensions often acknowledge the importance of geometry in the construction of regularly repeating patterns, they often hesitate to use systems which appear to require knowledge of sophisticated geometry. This reticence is understandable, since the bulk of literature produced by experts concerned with two-dimensional geometry is all too often wrapped in unfamiliar symbols and obscure terminology. From the viewpoint of symmetry, probably the best and most accessible sources to aid the construction of regular repeating patterns are Schattschneider (1978 and 1986), Stevens (1984) and Horne (2000).

Whilst the authors contend, as noted previously, that a well-developed understanding of Euclidean geometry is of value to the design practitioner, it should be stressed that the use of only a few basic geometric principles can have rather dramatic results. Also, various methods or systems of designing can be developed by reference to combinations of one or more of the principles, concepts, constructions, comparative measures and ratios listed in the introduction. With this in mind, this paper presents an outline of a system of pattern construction, developed by the authors in parallel with various research projects associated with the development of the design theory curriculum at their educational institution.

In its simplest form, the system allows the designer to select one of three regular polygons associated with the Platonic tilings. These are the equilateral triangle, the square and the hexagon, the only polygons capable of tiling the plane (on their own) without gap or overlap. Working with a particular end use in mind and making reference to storyboards, mood boards or other sources which express particular shapes, forms, structures and color palettes, the designer should cut each polygon into three or more unequal parts (or tiles). Each tile should be colored (or textured) by reference to an appropriate color palette or mood board. Multiple copies are made of each tile. Copies may be diminished or enlarged versions of the original. Many of these procedures can be readily facilitated through the use of appropriate software (such as Adobe Illustrator). Paying particular regard to the principle that a minimum inventory of tiles (or modules) is capable of wide diversity, the designer can then systematically manipulate the tiles by reference to the four symmetry operations and also to the seventeen basic pattern structures. Collections of resultant designs are shown in Figures 7 to 9.

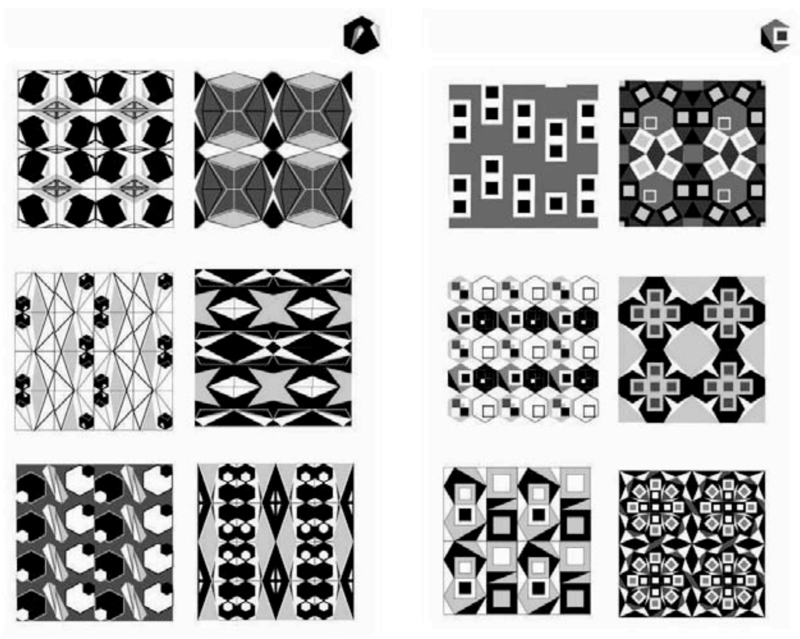

Figure 7: Illustration of a collection of modular tiling designs

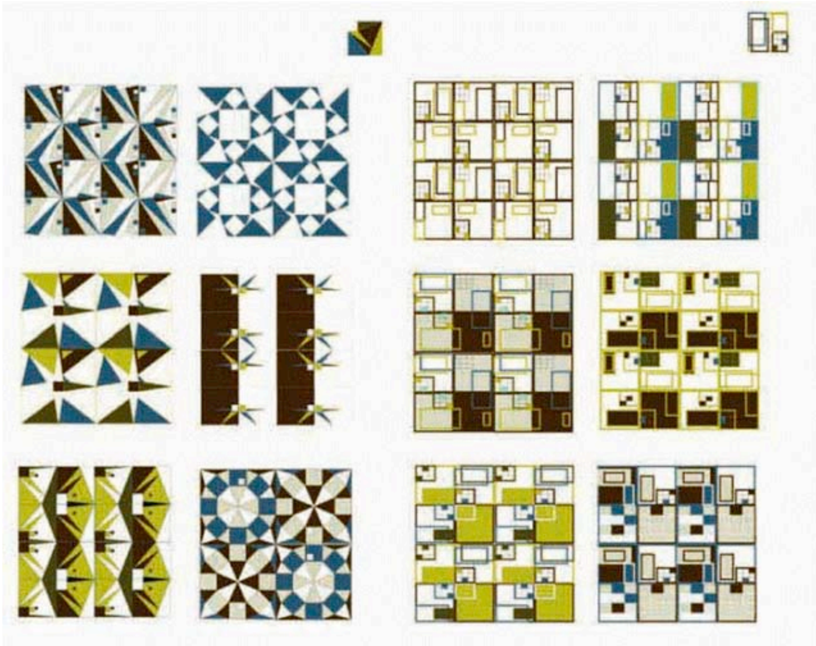

Figure 8: Illustration of a collection of modular tiling designs

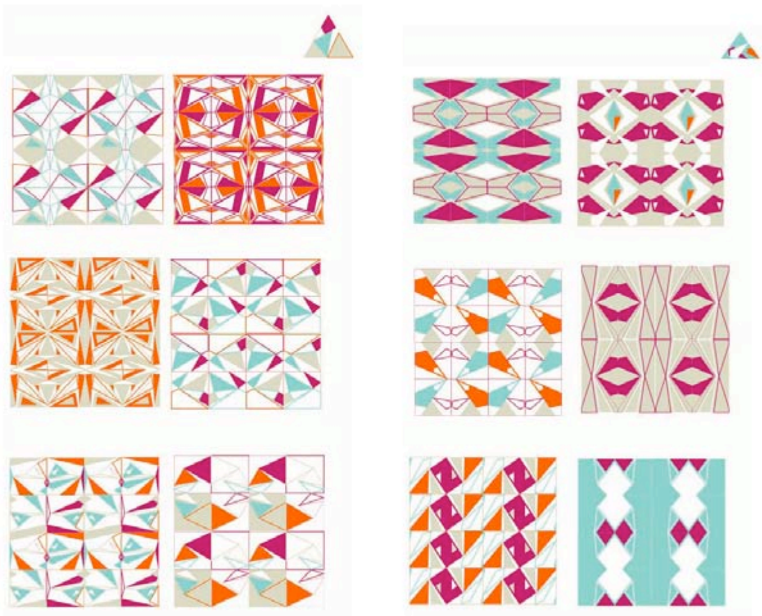

Figure 9: Illustration of a collection of modular tiling designs

5. IN CONCLUSION

Knowledge of many of the geometric principles, concepts and perspectives underlying structure and form in design can be sourced in ancient times and transcend the boundaries between art, design, science and engineering. An understanding of these can offer immense potential as problem solving design tools in the twenty-first century. This paper identified various geometric characteristics, principles, concepts, constructions, comparative measures and ratios which the authors maintain are of potential value to both the design practitioner and the design analyst. Particular attention was focused on geometric symmetry, and its use in design analysis and synthesis. Relevant empirical work, from the latter half of the twentieth century, was identified.

Symmetry classification relies on standardized units of measurement of a parameter which is fundamental to all decorated objects. The use of a classification system which lends itself to the precise ordering of data has ramifications of importance to anthropologists, archaeologists and design historians. Uniform classification makes possible the systematic study of data, the replication of results among researchers and the formation of plausible hypotheses and explanations. It is accepted, at least among the relevant group of researchers, that symmetry classification is capable of isolating an attribute which is culturally sensitive and, as such, offers the potential as a useful indicator of cultural adherence, continuity and change.

It is the contention of the authors that the relationship between geometry and culture is not expressed solely through geometric symmetry, but rather through a wide range of geometric measures, ratios or proportions (listed in the introduction); this is the subject of a recently initiated project.

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ACKNOWLEDGEMENT

The authors acknowledge the contribution of Miss Purnima Shah in the production of the illustrations presented in Figures 7 to 9.